

ever, the radii of the subsequent drops exceed the radius of the first drop by more than an order of magnitude, so that for $k \geq 2$ the term $\alpha_1(\tau)$ in the equations of the system (6) can be neglected as small. The same thing can be said in an investigation of the behavior of the system near $\tau \approx \tau_2$ and so forth, up to k which are not too large (the error grows in proportion to k). This indicates that the restrictions are reflected rather weakly in the final results for the radii $\alpha_{k+1}(\tau_k)$, where τ_k is determined from Eq. (11).

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LITERATURE CITED

1. Yu. P. Gupalo, A. D. Polyandin, and Yu. S. Ryazantsev, "On diffusion to a chain of drops (bubbles) at large Peclet numbers," *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 1 (1978).
2. A. D. Polyandin, "On the diffusional interaction of drops in a fluid," *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 2 (1978).
3. V. G. Levich, *Physicochemical Hydrodynamics*, Prentice-Hall (1962).
4. E. Wacholder and D. Weihs, "Slow motion of a fluid sphere in the vicinity of another sphere or a plane boundary," *Chem. Eng. Sci.*, 27, No. 10 (1972).
5. E. Ruston and G. A. Davies, "The slow motion of two fluid spheres along their line of centers," *Appl. Sci. Res.*, 28, Nos. 1-2 (1973).
6. L. D. Reed and F. A. Morrison, "The slow motion of two touching fluid spheres along their line of centers," *Int. J. Multiphase Flow*, 1, No. 4 (1974).

DRIPPING OF A LIQUID FROM A POINT

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The drop discharge of a liquid under the action of gravity is frequently encountered in nature, and is used in technology, e.g., in medicine droppers. In a slow drop discharge the drops formed are of approximately the same size. The formation of drops in the discharge of a liquid from a vertical stationary capillary has been investigated [1, 2]. The dripping of a liquid from a point has received little attention.

The formation of drops in the slow dripping of a liquid from the point of a vertical stationary needle whose surface is well wetted by the liquid is shown schematically in Figs. 1 and 2. The liquid flows by gravity from the reservoir 1 through the annular slit 2, and wets the conical surface of the needle of height H . The thickness of the layer of liquid increases after it flows through the slit, but not uniformly; visual observation shows that the liquid does not begin to accumulate at the point, but at a certain height h above it, in the form of a bulging collar (Fig. 2a). The thickness of the collar increases, it drops down (Fig. 2b), and gradually takes the form of a drop (Fig. 2c) which descends further to the point itself (Fig. 2d) and then quickly drips off the point (Fig. 2e) and falls downward (Fig. 2f).

In order to explain this dripping process, we consider the pressure distribution resulting from the surface tension of the liquid within a thin layer on the surface of a conical needle.

Forces due to the surface tension σ act on an annular element of the surface of a conical film of height dz at a height z (Fig. 3). The resultant of the vertical components of these forces is

$$[2\pi(R + dR)\sigma - 2\pi R\sigma] \cos \varphi = 2\pi\sigma \cos \varphi dR.$$

The resultant of the vertical components of the forces on this element as a result of the pressure p is $pS \sin \varphi$, where $S = \pi dL(2R+dR)$ is the area of the surface of the annular element (frustum of the cone), or, neglecting second order quantities, $S = 2\pi R dR / \sin \varphi$.

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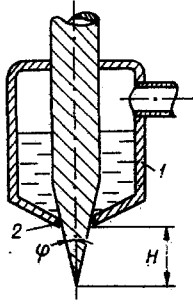


Fig. 1

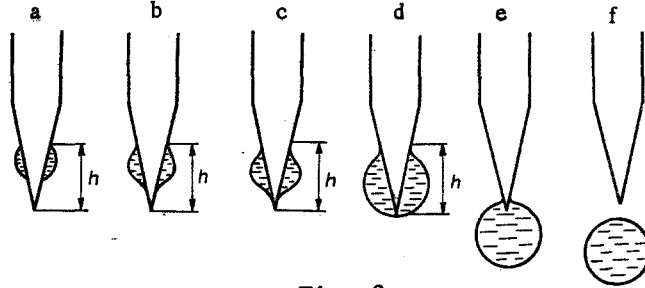


Fig. 2

Equating the two resultants, we obtain

$$p = \sigma \cos \varphi / R = \sigma \cos \varphi / z \operatorname{tg} \varphi, \quad (1)$$

i.e., the pressure within the conical film increases with decreasing z , and as $z \rightarrow 0$ (approaching the point of the needle) approaches infinity theoretically.

This is clearly the reason why drops do not begin to form at the point of the needle, but above it (Fig. 2a), and only as a drop is being formed, i.e., as the radius of curvature of the film is increasing, and consequently the pressure within it is decreasing, does the drop gradually descend to the point of the needle.

In order to estimate the size of drops formed in this way we assume that the motion is slow enough so that the viscous and inertial terms can be neglected in comparison with the gravitational force and the pressure forces in the equations of motion of a viscous liquid. We obtain

$$dp/dz + \rho g = 0. \quad (2)$$

We take this equation as the condition determining the height h at which a drop begins to form (Fig. 2a). According to (1)

$$dp/dz = -\sigma \cos \varphi / z^2 \operatorname{tg} \varphi$$

and Eq. (2) takes the form

$$\rho g = \sigma \cos \varphi / h^2 \operatorname{tg} \varphi.$$

The radius $R_1 = h \operatorname{tg} \varphi$ corresponding to the height h is

$$R_1 = \sqrt{\sigma \sin \varphi / \rho g}.$$

Equating the weight of a drop of diameter d_T to the surface tension force acting on it at a height h (Fig. 2d), we find the limiting diameter of a drop d_T which can remain in equilibrium from the expression

$$\frac{\pi d_T^3}{6} \rho g = 2\pi R_1 \sigma = 2\pi \sigma \sqrt{\frac{\sigma \sin \varphi}{\rho g}},$$

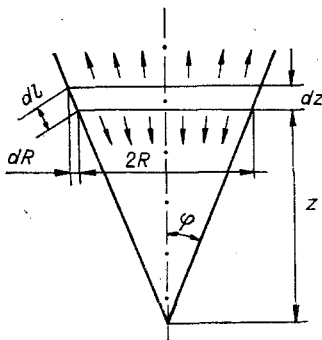


Fig. 3

TABLE 1

Liquid	Density ρ_{20° g/cm ³	Surface tension σ_{20° dynes/cm	Viscosity μ_{20° g/cm ² sec.
Mineral oil	0,87	30,5	0,6
Diesel fuel	0,84	29,6	0,04
Transformer oil	0,892	33,2	0,195
Lubricating oil	0,897	29,5	0,87

i.e.,

$$d_T = 2.3 (\sigma/\rho g)^{1/2} (\sin \varphi)^{1/6}. \quad (3)$$

We tested Eq. (3) by performing experiments in which we observed the dripping of mineral oil and diesel fuel from the point of a vertical brass needle which was well wetted by both liquids. For needle 1 the angle $\varphi = 7^\circ 50'$ and the height of the exposed part of the cone $H = 2.2$ cm; for needle 2 $\varphi = 12^\circ 50'$ and $H = 1.2$ cm. The characteristics of the liquids used are listed in Table 1.

Visual observations showed that the dripping of both liquids from both needles followed the pattern shown in Fig. 2.

Table 2 lists the values of the diameter of the drops falling from the point as measured by three different methods: collecting drops in an immersion medium and measuring them, catching drops on glass slides covered with silicone [3], weighing 100 drops. Table 2 also lists the theoretical values of the diameter of the drops d_T calculated by Eq. (3). The average experimental values of the diameter of the drops d_e determined by the three methods mentioned above turned out to be within 15.7% of one another. The values of d_e obtained by weighing were taken for comparison. The variation of d_T/d_e was within the limits 0.855-1.23* in all the experiments; on the average $d_T/d_e = 0.995$ with a mean-square deviation of 0.117. Thus, there is satisfactory agreement between the calculated and experimental results.

Equation (3) is valid only for very low flow rates Q of the liquid. As Q is increased, the frequency of drop formation increases, and in a certain range of values of Q the process changes character: as Q increases there are not only identical basic drops, but from the connecting necks between them more and more smaller satellite drops are formed; then the connecting necks between adjacent drops stop developing, and at a certain critical flow rate $Q = Q_C$ there is a transition from drop formation to stream-flow — the liquid is discharged from the capillary in a continuous stream rather than in the form of drops. The value of Q_C can be estimated from the expression [1, 2],

*The data in Table 2 shows that d_e increases somewhat with increasing flow rate of the liquid Q . This is not indicated by our approximate theory.

TABLE 2

No.	Type of dropper	Liquid	Flow rate of liquid Q , $\text{cm}^3/\text{sec.}$	Diam. of basic drops d_b , cm			Satellite drops			d_r / d_e
				Method of measurement		d , cm	neg/nba	G , %	d_r , cm	
				on silicone	immersion medium					
1			0,00025	0,27	0,28	0,27	None		0,31	1,45
2			0,0093	—	0,32	0,35	None		0,31	0,89
3			0,071	0,34	0,36	0,35	0,05--0,08	1	0,61	0,88
4	Needle 1		0,41	—	0,38	0,37	0,065	1	0,55	0,86
5		Mineral oil	0,006	—	—	0,30	None		0,31	1,03
6			0,018	—	—	0,33	—	1	—	0,94
7			0,042	—	—	0,36	—	1	—	0,88
8			0,0045	0,30	0,33	0,32	0,030	1	0,083	1,06
9	Needle 2		0,022	0,30	0,35	0,34	0,035	1	0,41	0,99
10			0,062	0,35	0,37	0,37	0,065	1	0,55	0,93
11	Needle 1	Diesel fuel	0,0017	0,28	0,25	0,25	0,059	1	1,25	1,23
12			0,030	0,30	—	0,28	—	—	—	1,42
13	Inclined plane	Mineral oil	0,05	0,51	0,51	0,50	0,14	3	6	
14			0,27	0,53	0,55	—	0,14	5	9	

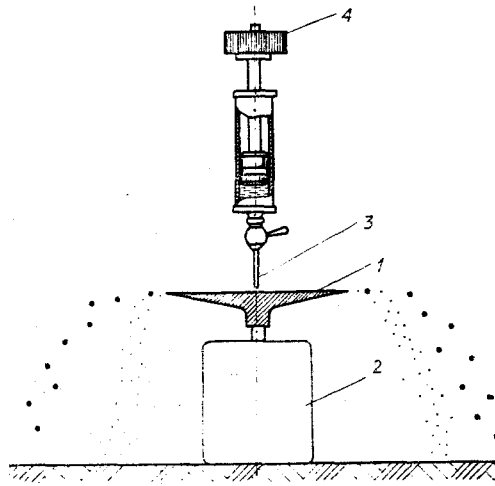


Fig. 4

$$Q_c = \frac{\pi}{6} K \sqrt{\frac{gd^5}{2}}, \quad (4)$$

where d is determined by Eq. (3), and the coefficient K takes account of the degree to which the calculated limit approximates the actual process.

Experiments show that even at very low flow rates of the liquid the formation of basic drops may be accompanied by the formation of smaller satellite drops which are produced in the break-up of the connecting necks between the basic drops. In the run off of drops from a point the neck connected to the drop at the instant of its separation from the point of the needle is thin, and consequently the satellite drops are small in size and few in number. Table 2 shows that when a liquid drops from a point, satellite drops are formed only at increased flow rates of the liquid, one satellite drop per basic drop. The satellite drops are 5-10 times smaller in diameter than the basic drops, and the fraction of the liquid contained in the satellite drops is only 0.1-1%. According to the data of Table 2, when liquid drips from the horizontal edge of an inclined plane [2], the satellite drops are relatively larger, only 3.5-4 times smaller than the basic drops, and there are more of them (2-5 satellite drops per basic drop). The fraction of the liquid contained in them (6-9%) is correspondingly larger also.

We turn to the break-up of a liquid by a rotating disk, a process analogous to the dripping of a liquid, but in this case the liquid experiences a centrifugal force in addition to gravity. Ordinarily gravity can be neglected in comparison with the centrifugal force.

In the break-up of a liquid by a rotating disk [2] the liquid is supplied to the center of the disk in a continuous stream, wets the surface, spreads out into a thin film, and for low flow rates (first atomization mode) is thrown from the edge of the disk in the form of basic drops of approximately the same size and smaller satellite drops.

At the present time both circular disks and disks with a notched periphery are used to atomize a liquid into drops of the same size. The atomization of a liquid by a circular disk is analogous to the dripping of a liquid from the lower edge of an inclined plane [2]; the analog of atomization by a notched disk is the dripping from a point.

The arrangement shown in Fig. 4 was used in experiments with circular and notched disks. The disk 1 is rotated by the electric motor 2. The liquid is supplied in a continuous stream to the center of the rotating disk from the needle of the injector 3 which is loaded with the weight 4. The drops formed settle onto silicone covered glass slides [3] and are measured under a microscope. The basic drops fall within a narrow annulus, since they are nearly identical and have nearly identical trajectories; the satellite drops settle mainly inside this annulus. By cutting out paper rings and sectors and weighing them before and after the atomization of a definite amount of liquid, the weight fraction of the basic drops and the satellite drops was determined for various flow rates of the liquid.

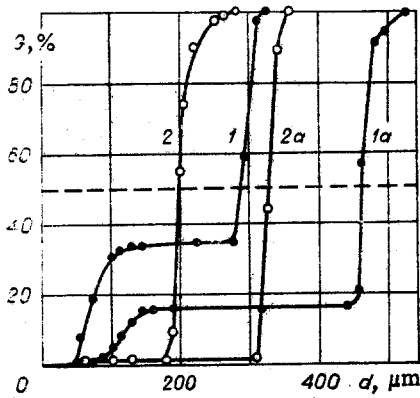


Fig. 5

Fig. 5 shows integral curves of the distribution of mineral oil drop sizes obtained with (1) a circular disk 80 mm in diameter and (2) a notched disk of the same diameter with 360 15° teeth uniformly spaced around the circumference; the angular velocity of the disk was 3000 and 1500 rpm (1a, 2a), and the flow rate of the oil was 0.1 cm³/sec. It is clear that in both regimes the atomization of the liquid occurs almost without the formation of small satellite drops: At 3000 rpm their total volume fraction is only ~ 1.5%, and at 1500 rpm ~ 1%. With the circular disk the volume fraction of the satellite drops was 34 and 16%, respectively. This agrees qualitatively with the results in Table 1 for dripping from a point and from an inclined plane.

To determine the theoretical diameter d_T of the basic drops obtained with the notched disk we transform Eq. (3) to a rotating disk by replacing the acceleration due to gravity g by the centrifugal acceleration $r\omega^2$, where r is the radius of the disk and ω is its angular velocity:

$$d_T = 2.3 \left(\frac{\sigma}{\rho r \omega^2} \right)^{1/2} (\sin \varphi)^{1/6}.$$

According to this equation $d_T = 174 \mu\text{m}$ at 3000 rpm and $348 \mu\text{m}$ at 1500 rpm, which are close to the experimental values $d_e = 200$ and $330 \mu\text{m}$.

The values of d_e for a circular disk are also close to the corresponding values calculated with the equations given in [2].

Figure 6 shows the dependence of the weight fraction of the basic drops E on the flow rate of the liquid Q for the circular and notched disks rotating at 3000 rpm. In the atomization of transformer oil (open circles), as already noted, the notched disk (1) forms appreciably fewer satellite drops than the circular disk (2), the transition from the first atomization mode to the second begins at a flow rate of the liquid $Q_{C1} = 0.9 \text{ cm}^3/\text{sec}$, $E = 70\%$, and ends at $Q_{C2} = 1.5 \text{ cm}^3/\text{sec}$, $E = 0$. With the circular disk the transition begins at $Q_{C1} = 0.52 \text{ cm}^3/\text{sec}$ and ends at $Q_{C2} = 1.03 \text{ cm}^3/\text{sec}$.

A qualitatively analogous pattern is observed for the atomization of lubricating oil (solid circles), at low flow rates of the liquid (first mode, $Q \leq 0.1 \text{ cm}^3/\text{sec}$) the number of satellite drops is several times smaller with the notched disk than with the circular disk, and the flow rates of the liquid Q_{C1} and Q_{C2} corresponding to the transition to the second atomization mode are larger with the notched disk than with the circular disk.

The value of Q_C can be estimated from the relation

$$\frac{Q_C}{360} = \frac{\pi}{6} K \sqrt{\frac{r\omega^2 d^5}{2}}. \quad (5)$$

Equation (5) differs from (4) only in the replacement of g by $r\omega^2$.

A comparison of values calculated by this equation with experimental values of Q_{C1} gives $K = 0.269$ for transformer oil, and $K = 0.06$ for lubricating oil. For Q_{C2} the values of K are 0.452 and 0.3, respectively.

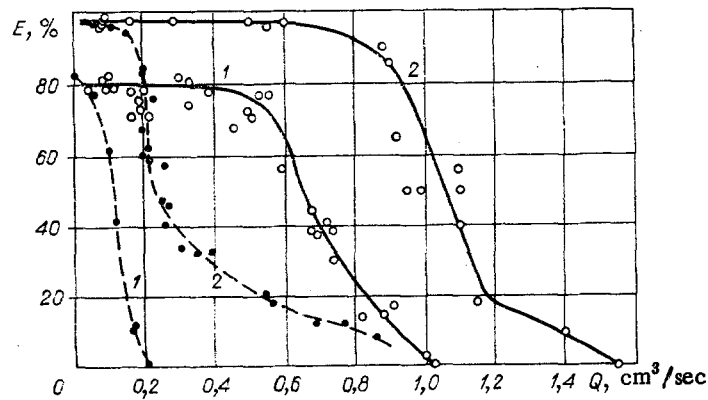


Fig. 6

For drop discharge from a stationary capillary an empirical formula is known for the dependence of the critical flow rate Q_c on the viscosity of the liquid μ : the critical flow rate Q_c decreases with increasing μ [1, 2]. The values of Q_c and K given above for transformer oil and lubricating oil indicate the existence of an analogous dependence of Q_c on μ for the atomization of a liquid by a notched disk.

A "monodispersed" atomizer is more suitable for technical applications if it forms smaller drops (other conditions being equal), a smaller relative number of satellite drops, and has a larger critical flow rate of the liquid Q_c . It is clear from the data presented that by all these criteria a notched disk is preferable to a circular disk.

It has been shown that Eq. (3), which was derived for the dripping of a liquid from a point, remains valid for the atomization of a liquid by a notched disk when g is replaced by ω^2 . This confirms the analogy between these two processes.

LITERATURE CITED

1. V. F. Duskii and N. V. Nikitin, "Transition from droplet flow to stream flow in a liquid," *Prikl Zh. Mekh. Tekh. Fiz.*, No. 5 (1974).
2. V. F. Duskii, N. V. Nikitin, and M. S. Sokolov, *Monodispersed Aerosols* [in Russian], Nauka, Moscow (1975).
3. Z. M. Yuzhnyi, "The microscopic determination of the size of drops of oily mists," *Kolloidn. Zh.*, 20 (1956).